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Learning by Failing: A Simple VaR Buffer

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We study in this article the problem of model risk in VaR computations and document a procedure for correcting the bias due to specification and estimation errors. This practical method consists of "learning from model mistakes", since it dynamically relies on an adjustment of the VaR estimates – based on a back-testing framework – such as the frequency of past VaR exceptions always matches the expected probability. We finally show that integrating the model risk into the VaR computations implies a substantial minimum correction to the order of 10–40% of VaR levels.

JEL classification: C14, C50, G11, G32.

I. INTRODUCTION

The recent worldwide financial crisis has dramatically revealed that risk management pursued by financial institutions is far from optimal. This paper proposes to illustrate an economic evaluation of the impact of model uncertainty on Value-at-Risk estimates based on a back-testing framework.

The VaR are used in asset management policies as well as micro-prudential regulations in both Banking (Basel II) and Insurance (Solvency II). This extreme risk measure serves to fix the required capital (Pillar I of Basel II regulation) and to monitor the risk by means of internal risk models (Pillar II of Basel II regulation). Risk estimates are thus used to determine capital requirements and associated capital costs of financial institutions, depending in part on the ex post quality of the recent VaR forecasts.

Hence, the amendment to the initial Basel Accord (BCBS, 1996) was designed to encourage and reward institutions for superior risk management systems. A back-testing procedure, that compares actual returns with the corresponding VaR forecasts, was introduced to assess the quality of the internal models. The objective was to monitor the frequency of so-called "exceptions" when realized losses exceed estimated VaR.

Therefore, appropriately constructed accurate risk measures, in particular those robust to model risks, are of paramount practical importance. Methods for the quantification of this type of risk are not nearly as well developed as methods for measuring market risk given a model, and the view is widely held that better methods to deal with model risk are essential to improve risk management.

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Table 1: Asymptotic Gaussian *versus* Small Sample Estimated Gaussian Quantiles

Probability Levels	Mean Est. VaR	Perfect VaR	Mean Bias	Min Bias	Max Bias
$\alpha = 95.00\%$	-29.49%	-29.49%	$.00\%$	-7.93%	7.24%
$\alpha = 99.00\%$	-41.88%	-41.88%	$.00\%$	-9.92%	9.17%
$\alpha = 99.50\%$	-46.41%	-46.41%	$.00\%$	-12.45%	10.16%

Source: These statistics were computed with the results on 100,000 simulated series of 250 daily returns according to a specific realistic Gaussian DGP and using an annualized Gaussian VaR. The columns represent, respectively, the average Estimated VaR with estimation errors and the average-minimum-maximum of the adjustment term for all samples. *Per* convention, a negative adjustment term indicates that the Estimated VaR (negative return) should be more conservative (more negative).

Furthermore, the Basel III committee has recently proposed that financial institutions assess their own model risk (BCBS, 2009). However, the model risk, whilst already studied in the case of specific price processes (e.g., Cont, 2006), is not yet practically taken into account in the building of risk models by the industry.[1]

But let us first grasp the intuition of our approach, in the simplest traditional framework based on the normal Brownian motion paradigm. The following Table 1 provides an illustration of the estimation risk within the classical Gaussian case, where, the asymptotic quantiles (simply assuming that returns are Gaussian) are compared to the small finite sample ones, computed in 100,000 trials with 250 observations (as demanded by regulators) of a realistic simulation (based on estimations of the first two realized moments of returns on the DJIA in USD, in the period from the 1st January, 1900 to the 15th October, 2010). The perfect (asymptotic) VaR, the average estimated VaR as well as the average-minimum-maximum of the errors (i.e., differences between asymptotic and estimated quantiles) are therein presented for three levels of probability. In the worst cases, the mean and volatility appear to be, respectively, severely over- and under-estimated within the small sample. This precise situation corresponds to a realization of an occasional quiet and stable up-trend, as seen sometimes in “bubble” periods in financial markets. We clearly see here that the estimation bias can represent up to 25% or so of the mean estimated VaR. Whereas existing works suggest that the estimation risk is of second-order importance when compared to the model mis-specification (Berkowitz, 2001) or data contamination (Frésard et al., 2011), this simple illustration shows, however, how important the model risk of risk models can be. In our previous elementary illustration within the classical paradigm, we can easily calibrate, in simulations, a correction based on some severe errors (adding, for

[1] Only a few recent papers (e.g., Kerkhof et al., 2010; Gouriéroux and Zakoïan, 2012; Breuer and Csiszár, 2012-a) aim to take model risk into account in the computation of risk measures.

instance in our case, an extra 25% buffer to the estimated normal quantile). But what can a risk manager do in real situations when he does not know for sure the "true" process behind, or the risk factor that is going to hit the market in the near future?

Our first aim is to gauge the importance of model risk in a risk management context, when adding other types of model risk, such as specification, data measurement, manipulation, liquidity (see Derman, 1996), in a more realistic context (outside the Gaussian paradigm) that deals with the main salient features of financial returns.

Our second aim is also to evaluate a simple strategy in order to at first confirm, with real data, our quantitative assessment of the value of model risk given in the previous controlled experiment with specific processes and, secondly, to propose a systematic approach to approximate model risk in risk models. This strategy merely consists of "learning from the model mistakes", since it dynamically relies on an adjustment of the VaR estimates (defining a buffer to be added to the estimated VaR). The size of correction is thus calibrated thanks to the back-testing framework of regulatory authorities – in such a way that the frequency of past VaR exceptions always matches the expected probability.

The outline of the paper is as follows. Section II provides a brief survey on model risk literature. Section III defines and illustrates the model risk in VaR estimates in an extensive realistic simulation exercise. Section IV presents our practical approach for calibrating adjusted empirical VaRs that deal with the model risk, when applied to a real financial series. Section V concludes.

II. ABOUT LITERATURE ON MODEL RISK

Our work is linked to various bodies of research and our aim is to contribute the general literature on model risks. While our approach is similar "in spirit" to the Bayesian literature on risk forecasting, our objective is to adjust extreme quantiles (such as VaR) in the context of financial risks to account for model uncertainty.

Generally speaking, model risks ultimately refer to inaccurate measures of a variable of interest; relatively small changes in the estimation procedure (model, method or sample) can modify the magnitude and even the sign of some important decision variables. Several sources can be at the origin of such model error, such as model misspecifications (e.g., Breuer et al., 2012) and estimation troubles (e.g., Lönnbark, 2010) – that are the main risks studied in the literature, as well as from data contamination (Frésard et al., 2011) or liquidity drastic shortages (see Derman, 1996, for a gentle typology).

As the first model risk, the one of estimation indeed occurs in every estimation process; it is the risk associated with inaccurate estimations of parameters, due to the estimator quality and/or limited sample of data (past and/or future), and/or noise in the data. This estimation risk is the most discussed in the literature (see for instance Gibson et al., 1999; and Talay and Zheng, 2002). Pritsker (1997) is one of the first to discuss the estimation risk for VaR in the identically and independently distributed return setting (see also, more recently, Inui and Kijima, 2005, with the same setting with the Expected Shortfall). Lönnbark (2010) quantifies the uncertainty due to the estimation when forecasting multiple period VaR.

Estimation risk when forecasting risk with dynamic models has also been considered by Berkowitz and O'Brien (2002), focusing on the too conservative VaR (at the time of publication), as well as Figlewski (2004) who examines the effect of VaR estimation errors based on simulations. Also, Bao and Ullah (2004) study the error in VaR estimates due, more specifically, to misspecified error distribution based on ARCH(1) models. Christoffersen and Gonçalves (2005) evaluate the precision of common models and quantify the magnitude of the estimation error by constructing confidence bands around the quantile forecasts. Chan et al. (2007) also propose to construct a confidence interval for the extreme conditional quantiles based on GARCH models with heavy-tailed innovations. More recently, Escanciano and Olmo (2009, 2010 and 2011) propose to correct the (bootstrapped) critical values in standard backtests on VaR to assess the estimation risk. Our work also directly relates to the recent works of Gagliardini et al. (2010), who propose Estimation and Granularity adjustments for VaR, and Gouriéroux and Zakoïan (2012), who propose a new estimator of the VaR that allows the forecaster to make a joint treatment of theoretical and estimation risks.

More generally, our work is also very similar in spirit to the work of West (1996), who discusses when and how to adjust critical values for tests of predictive ability in order to take parameter estimation uncertainty into account. Recent works extend this approach in a Bayesian VaR framework, trying to solve the so-called "Capital Charge Puzzle" when advocating that capital requirements were not so large (at the time) when parameter uncertainty was taken into account (Pollard, 2007), and that dealing with uncertainty allows the risk manager to recover a very conservative VaR (see Hoogerheide and van Dijk, 2010; Chen et al., 2012), most of the time even more prudent than the supremum of a group of classical models of VaR (Casarin et al., 2012). In a sense, our proposal is also close to the method proposed by Hartz et al. (2006) who correct the Gaussian VaR with re-sampling techniques in order to be as close as possible to the perfect model in terms of frequency of occurrences of large errors.

However, we will hereafter adopt a rather different approach, even if similar, trying to explicitly link a correction for uncertainty to the simple test that is imposed by regulators; expressing furthermore the required correction as an additive buffer for model risk as called for by the regulation. In other words, it will be easy for the risk manager in our setting to translate the correction of VaR into penalty avoidances expressed in euros. Moreover, we deal in one shot with all potential model errors (estimation included, but not only) and we will accordingly adjust levels of risk measures. While it is of course possible and useful to investigate the effect of parameter variations on risk measures that are computed in a particular parametric framework, as for instance in Bongaerts and Charlier (2009), our aim here is to explicitly consider model risk as a separate risk factor (i.e., independent of a special data generating process, DGP).

The objective of this paper is, finally, similar to the one of Kerkhof et al. (2010), who first propose a procedure to take model risk into account in the computation of capital reserves calibrated on the backtesting framework of the regulators, as well as Alexander and Sarabia (2012), who develop a methodology for quantifying model risk in quantile risk estimates based on a maximum entropy criterion. Also, in the same vein, Breuer and Csiszr (2012-a and 2012-b) and Breuer et al. (2012) propose to quantify model risk as the largest loss from on a distribution which is at a reasonable (Mahalanobis or Kullback-Leibler) distance to a reference density.

In this general context of related literature, our main contribution herein thus consists of proposing a simple framework to compute risk measures robust to the main model risks, based on an incremental buffer assessing the main valuable properties of risk models.

III. MODEL RISK AND VAR COMPUTATIONS

We first illustrate, in a realistic simulation framework, the model risk of VaR estimates, which is here defined as the consequence of two types of error due to a model misspecification and a parameter estimation uncertainty. Various VaR computation methods do indeed exist in the literature, from non-parametric, semi-parametric and parametric approaches (e.g., Engle and Manganelli, 2001; Christoffersen, 2009). However, the historical-simulated VaR computation is still the one most used by practitioners (Christoffersen and Gonçalves, 2005; Pérignon and Smith, 2010) and thus will serve as the reference throughout this article[2].

Table 2 presents the estimated VaR as well as the mean, minimum and maximum errors on these quantile estimates. Errors are defined by the differences between the "true" asymptotic VaR (based on simulated DGP) and the imperfect historical-simulated estimated VaR (because the latter are only approximately specified and estimated with a limited data sample). Three different rolling time-windows (from 250 to 750 days in Panels A, B and C) and several levels of probability confidence thresholds (three rows for each Panel) are considered. The results are presented for three DGP for the underlying stock price with various intensities of jumps (Brownian, Lévy and Hawkes[3]). The Brownian motion case is a standard in finance, whilst a Lévy process allows us to take into account (negative) discontinuities in prices (see Prigent, 2007; for traditional applications in quantitative finance of such a process).

[2]For the sake of completeness, we also use in this setting a group of other well-known methodologies for computing estimated quantiles, from GARCH to extreme density VaR (available to readers on demand), with, at the end, the same qualitative result regarding the overall importance of model risk; furthermore, no model was shown to be clearly superior on a statistical basis (i.e., required corrections being of the same order for best models), even if, as our intuition tells us, final corrections appear to be much lower for dynamic VaR models. We then first choose herein to stay with the industry benchmark to be used as the reference along this note, just for highlighting the magnitude of model risk in such a common approach.

[3]See e.g., Applebaum (2004), Bowsher, (2007), Prigent (2007), Aït-Sahalia et al. (2010), Bacry et al. (2011), for Lévy's and Hawkes' process definitions and uses in finance.

Table 2: Estimated annualized VaR and model-risk errors (in%)

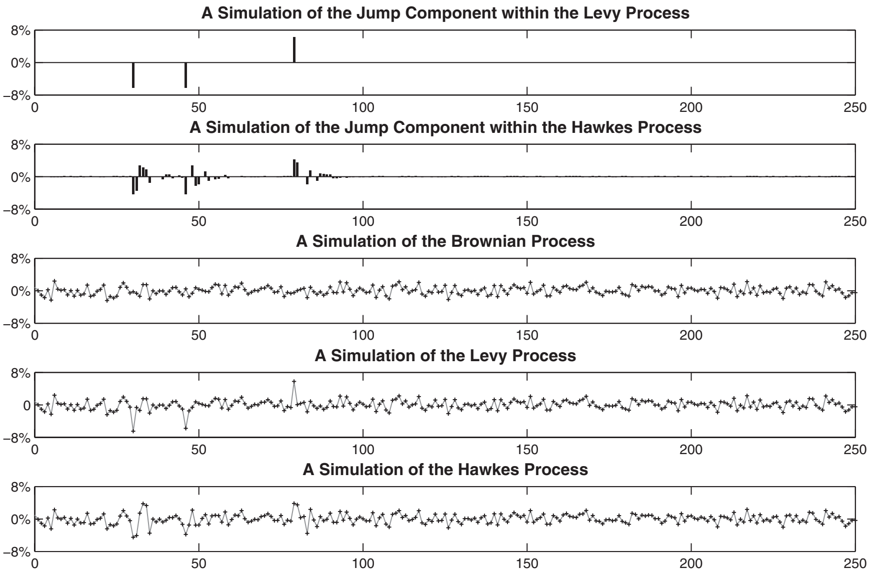
Three price processes of the asset returns are considered below, such as for $t = [1, \ldots, T]$ *and* $p = [1, 2, 3]$:

$$dS_t = S_t(\mu dt + \sigma dW_t + J_t^p dN_t) \qquad \text{with}: \begin{cases} J_t^1 = 0 & (1.\ \textit{Brownian}) \\ J_t^2 = \lambda_2\ \exp(-\lambda_2 t) & (2.\ \textit{Lévy}) \\ J_t^3 = \lambda_3 + \beta \exp[-\gamma(t-s)] & (3.\ \textit{Hawkes}) \end{cases}$$

where S_t is the price of the asset at time t, W_t is a standard Brownian motion, independent from the Poisson process N_t, governing the jumps of various intensities J_t^p (null, constant or time-varying according to the process p), defined by parameters, λ_2, λ_3, β and γ, which are some positive constants, with s (in case 3) the date of the last observed jump.

Processes	1. Brownian				2. Lévy				3. Hawkes			
VaR and Error (in%):	Mean Estimated VaR	Mean VaR Error	Min VaR Error	Max VaR Error	Mean Estimated VaR	Mean VaR Error	Min VaR Error	Max VaR Error	Mean Estimated VaR	Mean VaR Error	Min VaR Error	Max VaR Error
Probability	Panel A. One-year Rolling Window Calibration ($T = 250$)											
95.00%	−24.78	−0.06	−8.69	10.16	−25.18	2.49	−6.53	15.28	−27.33	3.51	−6.52	25.25
99.00%	−35.74	−0.11	−14.21	20.70	−39.07	5.07	−12.35	101.73	−44.78	7.47	−14.64	61.16
99.50%	−39.95	0.09	−16.04	28.92	−54.26	14.90	−14.60	122.56	−54.16	9.14	−19.50	65.39
	Panel B. Two-year Rolling Window Calibration ($T = 500$)											
95.00%	−24.81	−0.03	−6.25	7.34	−25.20	2.51	−3.63	10.29	−26.87	3.05	−4.77	15.03
99.00%	−35.85	0.00	−10.04	14.45	−38.18	4.19	−7.73	75.61	−43.35	6.04	−11.03	43.80
99.50%	−39.81	−0.06	−12.66	19.97	−49.60	10.24	−12.15	102.05	−52.28	7.26	−16.88	54.31
	Panel C. Three-year Rolling Window Calibration ($T = 750$)											
95.00%	−24.82	−0.03	−5.56	5.99	−25.21	2.52	−3.26	8.63	−26.72	2.89	−3.43	12.40
99.00%	−35.86	0.01	−8.54	11.29	−37.86	3.87	−6.05	66.22	−42.69	5.37	−8.34	39.40
99.50%	−39.90	0.03	−10.61	14.57	−48.38	9.02	−8.94	93.76	−51.85	6.84	−13.83	45.78

Source: simulations by the authors. Errors are defined as the difference between the "true" asymptotic simulated VaR and the Estimated VaR. These statistics were computed with a series of 250,000 simulated daily returns with specific DGP (1. Brownian, 2. Lévy and 3. Hawkes), averaging the parameters estimated in Aït-Sahalia et al. (2010, Table 5, *i.e.* $\beta = 41.66\%$, $\lambda_3 = 1.20\%$ and $\gamma = 22.22\%$), and *ex post* recalibrated for sharing the same first two moments (*i.e.* $\mu = .12\%$ and $\sigma = 1.02\%$) and the same mean jump intensity (for the two last processes such as $\bar{J}_t^2 = \bar{J}_t^3$ – which leads after rescaling here, for instance, to an intensity of the Levy such as: $\lambda_2 = 1.06\%$). *Per* convention, a negative adjustment term in the table indicates that the Estimated VaR (negative return) should be more conservative (more negative).



Source: Simulations by the authors. These five illustrative figures were computed with one series of 250 simulated daily returns using specific DGP (1.Brownian, 2. Lévy and 3. Hawkes), averaging the parameters estimated in Aït-Sahalia et al. (2010, table 5) for major stock markets. For this drawing, the annualized 99.50% estimated VaR are the same with the three DGP because the extreme points are the same within these three simulations, but the 99.00% and 95.00% annualized VaR are different. These five plots illustrate respectively: the behaviour of the jumps within the Lévy process and then within the Hawkes process; the three last plots illustrate a simulation of the returns using a Brownian, a Lévy and a Hawkes process (see Table 2 below for precise definitions of processes in our context).

Figure 1: An Illustration of Simulated Brownian, Lévy and Hawkes' Processes for a Series of 250 Returns.

When, finally, auto-regressivity in jumps is considered in Hawkes' (1971) case, the process is able to reproduce main documented characteristics of the financial returns such as sudden shocks, self-excitements, regimes, heteroskedasticity, clustering of extremes, asymmetry and excess kurtosis, as illustrated in Figure 1, with examples of a limited series of 250 returns following the three underlying processes that are considered herein.

As reported in Table 2, and as intuitively expected, the estimated VaR is an increasing function of the confidence level and the presence of jumps in the process (Lévy's and Hawkes' cases). For a large number of trials, the mean bias of the historical-simulated method is quite small (inferior to 1% in relative terms) in the Brownian case, and rather insensitive to the number of data points in the sample. By contrast, this mean bias is rather large when jumps are considered (with an amplitude of 10% to 27% in relative terms).[4]

Moreover, the observed range of potential relative errors (the difference between the maximum and minimum estimated errors divided by the estimated VaR) is substantial in our experiments, representing between around 40% of the VaR levels in the best case (for the simple Brownian DGP over the longer sample) to as high as 290% in the worst case scenario (for the Lévy DGP over the shorter sample). Furthermore, the potential relative under-estimation of the "true" VaR (too aggressive estimated VaR) is, in the main, large (ranging from 13% to 40%, depending on the sample length and the quantile considered[5]). These results suggest that the historical-simulated VaR should be corrected when safely taking into account the riskiness of risk models.

IV. A SIMPLE PROCEDURE FOR ADJUSTING ESTIMATED VAR

We now present herein a simple procedure to calibrate a correction of VaR estimates to account for the impact of the model errors, which could be applied by a risk manager with real data when the "true" DGP is unknown. This procedure is based on the "traffic light" test developed by the Basel Committee. The regulatory back-testing process is carried out by comparing the last 250 daily 99% VaR estimates with corresponding daily trading outcomes.

The regulatory framework is based on the unconditional coverage test (Kupiec, 1995) and then only focuses, at the time, on the proportion of failures. This test refers to the so-called "hit variable" associated to the *ex post* observation of estimated VaR violations at the threshold α and time t, denoted $I_t^{EVaR}(\alpha)$, which is defined as:

$$I_t^{EVaR(.)}(\alpha) = \begin{cases} 1 & \text{if } r_t < -EVaR(P,\alpha)_{t-1} \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $EVaR(.)$ is the Estimated VaR on a portfolio P at a threshold α, and r_t is the return on a portfolio P at time t, with $t = [1, \dots, T]$.

If we assume that the $I_t^{EVaR}(\cdot)$ variables are Independently and Identically Distributed, then, under the unconditional coverage *hypothesis* (Kupiec, 1995), the total number of VaR exceptions (Cumulated Hits) follows a Binomial distribution (Christoffersen, 1998), denoted $B(T,\alpha)$, such as:

$$Hit_T^{EVaR(.)} = \sum_{t=1}^{T} I_t^{EVaR(.)}(\alpha) \underset{T\to+\infty}{\sim>} B(T,\alpha) \tag{2}$$

[4]The relative error of 27% corresponds to the probability 99.50% with a window of 250 days for the Lévy DGP (i.e., 14.90 out of −54.26).

[5]The relative error of 40% is related to a probability of 99.50% in a 250 day window of Brownian returns (i.e., −16.04 out of −39.95).

A Perfect VaR (not too aggressive, but not too confident) in the sense of this regulatory rule, is such that it provides a sequence of VaR denoted $VaR(.)^*$ (*i.e.* all $\{VaR(P,\alpha)_t^*\}$ for $t = [0, T]$), that respects:

$$\begin{cases} T^{-1} Hit_T^{VaR(.)^*} < \alpha \\ T^{-1}\left[Hit_T^{VaR(.)^*} + 1\right] \geq \alpha, \end{cases} \tag{3}$$

where $Hit_T^{VaR(.)*}(\cdot)$ is the cumulated hit variable associated to the $VaR(.)^*$.

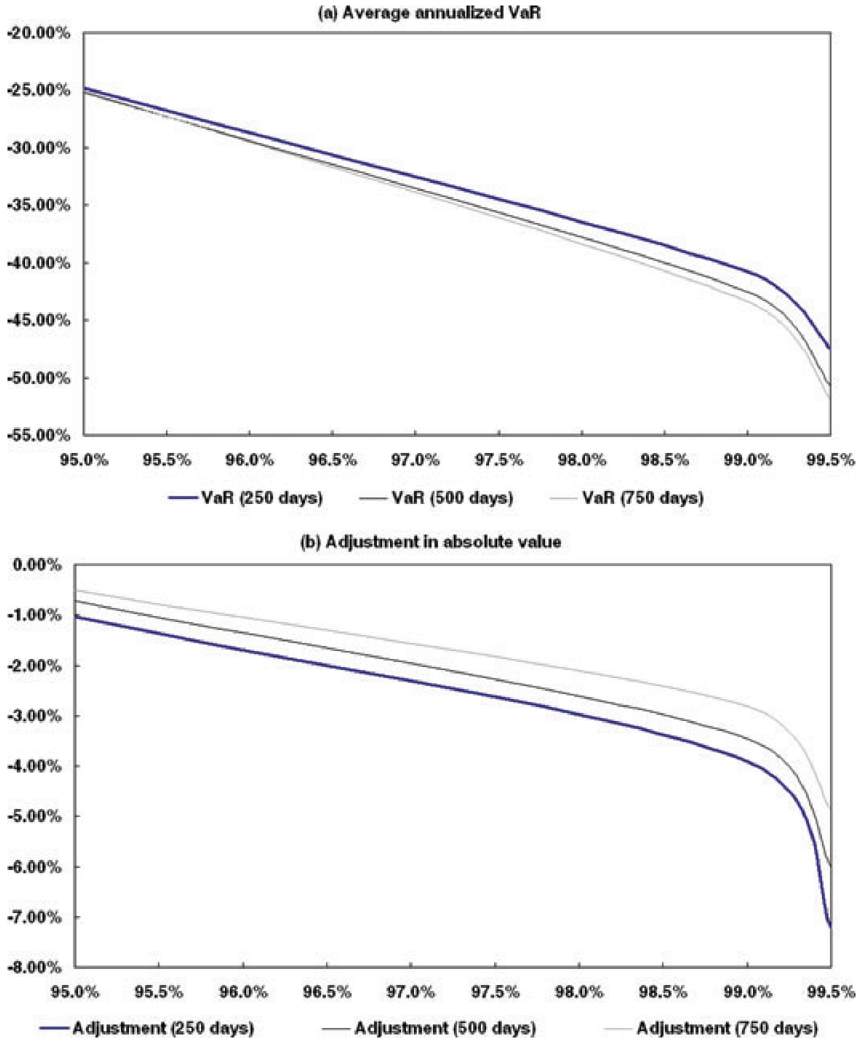
In other words, since the estimated VaR and the bounding range of violations are known, we now have to search, amongst all possibilities, for the minimum (unconditional) adjustment that allows us to recover a corrected estimated VaR that respects condition (3) over the whole sample, *i.e.:*

$$\begin{aligned} &\underline{adj}(P,\alpha) = q^* = \underset{q^* \in IR}{ArgMax}\{VaR(P,\alpha)_t^*\} \\ &s.t.: \\ &\begin{cases} T^{-1} Hit_T^{VaR(.)^*} < \alpha \\ T^{-1}\left[Hit_T^{VaR(.)^*} + 1\right] \geq \alpha, \end{cases} \\ &\text{with}: \\ &VaR(P,\alpha)_t^* = EVaR(P,\alpha)_t + q^*. \end{aligned} \tag{4}$$

In the following, we consider a long data set, containing rich information about the conditional and unconditional distributions of returns, which consists of daily returns on the Dow Jones Industrial Average (DJIA) index from the 1st January, 1900 to the 15th October, 2010.[6] This long data set is frequently used in empirical studies since it offers a variety of behaviors of volatility and extreme returns (e.g., Sullivan et al., 2000; Chernov et al., 2003; Zumbach and Finger, 2010) and is long enough for providing some robustness cautions.

Figure 2 represents the minimum adjustments (absolute errors) to be applied to estimated VaR, denoted $\underline{q}^*$, as solutions of the optimization program (4), for one-year (two-year and three-year) historical-simulated VaR computed on the DJIA over more than one century.

[6]Note here that the procedure can easily be applied to large multivariate portfolios of (non-linear) assets as in any other traditional *ex ante* risk management strategy, since it only requires actual asset weights and a (long) history of net asset values (altogether with the valid valuation *formula* for non-linear pay-offs if any). Please note here that the very same exercise of calibrating the buffer for corrections of VaR has been realized on other stock price and diversified portfolio series (with or without short-sale constraints), with – at the end – similar qualitative results on the size of the required corrections. Also, we use here a very long time-series to gauge the global validity of the approach, even if we generally do not have at our disposal such long series in real management practices. However, companion tests (available on request) show that less than one decade is necessary for reaching the final correction (even in the worst case *scenario*).



Source: Bloomberg; daily data of the DJIA index in USD from the 1st January, 1900 to the 15th October, 2010; computations by the authors. The first plot (on the left hand side) represents the non-adjusted average annualized VaR level. The minimal adjustment is represented in the second plot and is expressed in absolute value (on the right hand side). The minimal adjustment necessary to respect the hit ratio *criterion* is here considered as a *proxy* of the economic value of the model risk. The historical VaR is computed on a daily horizon as an annualized empirical quantile using respectively 1 year, 2 years and 3 years of past returns. Without any adjustment, the imperfect Estimated VaR is underestimated (too permissive) in each of these cases.

Figure 2: Minimum Model Risk Adjustments associated to Historical-simulated VaR.

In other words, it represents the minimal global constants that we should have added to the quantile estimates for having reached a VaR sequence that would have passed the Hit test on the full sample for a considered level of confidence. We observe that the historical-simulated error is quite significant for all quantiles (between -0.5% and -7% in absolute terms, i.e., 15% or so in relative terms) and significantly increases with the confidence level.

Besides, the smaller the estimation period, the more important the adjustment (both in absolute and relative terms). This *phenomenon* can be explained by the fact that using larger estimation periods is more likely to take into consideration extreme realizations and crisis episodes within the considered sample.

V. CONCLUSION

Following Kerkhof et al. (2010) who have recently proposed a similar bias estimation correction, this note illustrates a practical method to incorporate model risk into risk measure estimates, by adjusting the estimated VaR according to the frequency of past exceptions. This VaR adjustment allows the risk manager a joint treatment of theoretical and estimation risks, taking into account their possible dependence.

We first show in simulations that the model risk can represent a significant part of the risk measure and secondly, with real data, that the required correction may be substantial (of the order of $10-40\%$ of VaR levels in an extensive simulation, and in the range of 1–15% for a real risky stock market index).

We have here focused on studying an historical-simulation VaR (since most employed in financial institutions), a small data sample issue (because mere tests use 250 observations) and the frequency rule (as defined by regulators). Since we have now shown that model risk may play a decisive role in a normal risk management strategy in such a traditional context, our work can be extended in several ways in the future. First, the *criterion* we used here to quantitatively enforce the definition of VaR, based on a prior *in-sample* estimation and used in *out-of-sample* computations, could be generalized and complemented using extra tests of the good qualities of a VaR model, such as the independence and the limited magnitude of violations. Secondly, it may be interesting to use the metric of correction in order to evaluate the efficiency of existing various methodologies for computing VaR (the lower the correction, the better the model – see Boucher et al., 2013). Finally, a complementary study of the sensitivity of the proposed methodology to sample periods for several types of (non-linear) portfolios should be worthy.

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